Design of minimum water-loss canal sections

Calcul de sections de canal minimisant la perte d’eau

PRABHATA K. SWAMEE, Prof. of Civ. Engrg., Univ. of Roorkee, Roorkee 247 667, India

GOVINDA C. MISHRA, Scientist, Nat. Inst. of Hydrology, Roorkee 247 667, India

BHAGU R. CHAHAR, Asst. Prof. in Civ. Engrg., M.B.M. Engrg. College, JNV Univ., Jodhpur 342 011, India

SUMMARY

The canal water losses constitute of seepage and evaporation losses. Whereas seepage loss depends on the channel geometry, evaporation loss is proportional to the area of free surface. On account of complexities of analysis, the design of minimum water loss section has not been attempted as yet. In this investigation explicit equations for the design variables of minimum water loss sections for triangular, rectangular, and trapezoidal canals have been obtained using non-linear optimization technique. The proposed equations along with tabulated section shape parameters facilitate easy design of the minimum water loss section and computation of water loss from the section without going through the conventional and cumbersome trial and error method. A design example has been included to demonstrate the simplicity of the method.

RÉSUMÉ

Les pertes d’eau en canal sont dues aux infiltrations et aux pertes par évaporation. Alors que les pertes par infiltration dépendent de la géométrie du canal, l’évaporation est proportionnelle à l’aire de la surface libre. Compte tenu des complexités de l’analyse, l’étude d’une section minimisant la perte d’eau n’avait jusqu’à présent pas été tentée. Dans la présente investigation, des équations explicites ont été obtenues pour les variables de calcul des sections de perte d’eau minimale, dans les cas de canaux de section triangulaire, rectangulaire et trapezoïdal, en utilisant une technique d’optimisation non linéaire. Les équations proposées, ainsi qu’une tabulation des paramètres relatifs aux formes des sections, facilite la recherche de la section de perte d’eau minimale et le calcul de cette perte, sans avoir recours à la méthode conventionnelle et fastidieuse d’essai erreur. Un exemple est présenté pour démontrer la simplicité de la méthode.

KEY WORDS

canal design, canals, hydraulic structures, optimal sections, seepage loss, evaporation loss, and uniform flow.

1. Introduction

The loss of water due to seepage and evaporation from irrigation canals constitutes a substantial part of the usable water. By the time the water reaches the field, more than half of the water supplied at the head of the canal is lost in seepage and evaporation [11]. Seepage loss is the major and the most important part of the total water loss [14]. The other part i.e. evaporation loss is important particularly in water scarce areas. Considerable part of flow may be lost from a network of canals by the way of evaporation in high evaporating conditions. This needs special consideration for a long channel carrying small discharge in arid regions. Thus, care must be taken in the design of such canals to account for evaporative losses along with seepage loss.

A review of literature reveals that though considerable work has been reported on the design of minimum area cross section, practically no work has been done on the minimum water loss canal sections. Swamee [13] has reviewed the existing literature on minimum area canal sections. In the present study using explicit equations for seepage loss [14], the evaporation equation for flowing channels [5], and the general resistance equation for open channel flow [12], minimum water loss sections have been obtained by applying non-linear optimization technique for triangular, rectangular, and trapezoidal canal sections.

2. Water Losses

Water losses are on account of seepage and evaporation.

2.1 Seepage Loss

Providing perfect lining can prevent seepage loss from canals but cracks in lining develop due to several reasons and performance of canal lining deteriorates with time. An examination of canals by Wachyan and Rushton [15] indicated that even with the greatest care the lining does not remain perfect. A well maintained canal with 99% perfect lining reduces seepage about 30-40% only [15]. Thus significant seepage losses occur from a canal even if it is lined. The seepage loss from canals is governed by hydraulic conductivity of the subsoil, canal geometry, and potential difference between the canal and the aquifer underneath which in turn depends on the initial and boundary conditions. Seepage losses are also influenced by clogging of the canal surfaces depending on the suspended sediment content of the water and on the grain size distribution of the suspended sediment particles. The clogging process can decrease the seepage discharge both through bottom and slopes. Thus the seepage loss can change within time and under certain conditions it can diminish. Therefore, the seepage loss can be higher at the beginning of the canal operation and can be lower after a few years of operation.

The seepage loss from a canal in an unconfined flow condition is finite and maximum when the potential difference is very large e.g. when the water table lies at very large depth. The steady seepage loss from an unlined or a cracked lined canal in a homogeneous and isotropic porous media, when water table is at very large depth, can be expressed as
\[ q_s = k_y F_s \]  

where \( q_s \) is seepage discharge per unit length of canal (m\(^2\)/s); \( k \) = coefficient of permeability (m); \( y_n \) = normal depth of flow in the canal (m); and \( F_s \) = seepage function (dimensionless), which is a function of channel geometry.

The seepage function can be estimated for different sets of specific conditions for a known canal dimensions \([6,8,10]\). The analytical form of these solutions, which contain improper integrals and unknown implicit state variables, are not convenient in estimating seepage from the existing canals and in designing canals considering seepage loss. These methods have been simplified using numerical methods for easy computation of seepage function by Swamee et al \[14\].

### 2.2 Evaporation Loss

Evaporation loss depends on (1) the supply of energy to provide the latent heat of vaporization and (2) the ability to transport the vapor away from the evaporating surface, which in turn depends on the wind velocity over the surface and the specific humidity gradient in the air above the water surface. A large number of equations for estimating evaporative rate are available in the literature. A review indicated that these equations fall into the following categories: (a) energy balance equations; (b) mass transfer equations; and (c) combinations of the two. Warnaka and Pochop \[16\] and Ikebuchi et al \[7\] compared the merits of various equations; (a) energy balance equations; (b) mass transfer equations for estimating evaporative rate are available in the literature etc. make the method unattractive. On the other hand, the mass transfer equations are most convenient and useful for determining seepage from the existing canals and in designing canals considering seepage loss. Once \( E \) is known the evaporation loss from a canal can be expressed as

\[ q_s = ET \]  

### 2.3 Total Water Loss

Adding (1) and (6) the total water loss \( q_w \) (m\(^2\)/s) was expressed as:

\[ q_w = ky_s F_s + ET \]  

Using Swamee et al \[14\] equations for \( F_s \), Eq. (7) for triangular channel section was reduced to

\[ q_w = ky_s \left[ (4\pi - \pi^2)^{1.3} + (2m)^{1.3} \right]^{0.77} + bE \]

where \( m = \) side slope. See Fig. 1(a). Similarly for rectangular section (7) was changed to

\[ q_w = ky_s \left[ (4\pi - \pi^2)^{0.77} + (b/y_s)^{0.77} \right]^{1.3} + bE \]

where \( b = \) bed width of the section. See Fig. 1(b). On the hand, for trapezoidal section [See Fig. 1(c)], (7) was reduced to

\[ q_w = ky_s \left[ (4\pi - \pi^2)^{1.3} + (2m)^{1.3} \right]^{0.77} + \left( b/y_s \right)^{1.3} + \left( b + 2my_s \right)E \]
3. Resistance Equation

Uniform open channel flow is governed by the resistance equation. The most commonly used resistance formula is Manning’s equation [2], which is applicable for rough turbulent flow, and in a limited bandwidth of relative roughness [3]. Relaxing these restrictions, Swamee [12] gave the following resistance equation:

\[ Q = -2.457A_s g R_0 S_{bs} \ln \left( \frac{e_{n}}{12R_s} + \frac{0.221v_s}{R_s^{1.5}} \right) \]  

(11)

where \( Q \) = canal discharge (m\(^3\)/s); \( A_s \) = flow area (m\(^2\)); \( g \) = gravitational acceleration (m/s\(^2\)); \( R_s \) = hydraulic radius (m) defined as the ratio of the flow area to the flow perimeter \( P \) (m); \( e_s \) = average roughness height of the canal lining (m); and \( v \) = kinematic viscosity of water (m\(^2\)/s). Similar to the case of resistance equation for pipe flow, (11) involves physically conceivable parameters \( e \) and \( v \).

4. Non-dimensionalization

Defining the length scale \( \lambda \) as

\[ \lambda = \left( \frac{Q^2}{gS_b} \right)^{0.2} \]

(12)

the following non dimensional variables were obtained:

\[ y_s = \frac{y_s}{\lambda}; \quad b_s = \frac{b}{\lambda}; \quad R_s = \frac{R}{\lambda}; \quad T_s = \frac{T}{\lambda}; \quad A_s = \frac{A}{\lambda^2} \]

(13a-e)

\[ e_s = \frac{e}{\lambda}; \quad v_s = \frac{\sqrt{\lambda}}{Q}; \quad E_s = \frac{E}{k}; \quad q_{n_s} = \frac{q_{n_s}}{k\lambda} \]

(13f-i)

\( E_s \) depends on the type of soil and the climatic conditions of the canal site. Using these non-dimensional parameters (7) reduced in non-dimensional water loss form as

\[ q_{n_s} = y_s F_s + E_s T_s \]

(14)

while (11) became the non-dimensional flow resistance equation

as

\[ 1 + 2.457A_s g R_0 \ln \left( \frac{e_{n}}{12R_s} + \frac{0.221v_s}{R_s^{1.5}} \right) = 0 \]

(15)

5. Optimization Algorithm

The problem of determination of optimal canal section shape was reduced to

Minimize  \( q_{n_s}^* = y_s F_s + E_s T_s \)

subject to  \( \phi = 1 + 2.457A_s g R_0 \ln \left( \frac{e_{n}}{12R_s} + \frac{0.221v_s}{R_s^{1.5}} \right) = 0 \)

(17)

where \( \phi \) = equality constraint function. The constrained optimization problem (16)-(17) was solved by minimizing the augmented function \( \psi \) given by

\[ \psi = q_{n_s}^* + \rho \phi^2 \]

(18)

where \( \rho \) = a penalty parameter. Adopting small \( \rho \) initially, (18) was minimized using grid search algorithm. Increasing \( \rho \) five-fold, the minimization was carried through various cycles until the optimization results stabilized.

6. Optimal Section Shapes

The optimization algorithm was applied on triangular, rectangular, and trapezoidal canal sections for a number of input variables varying in the ranges

\[ 10^{-6} \leq e_s \leq 10^{-3}; \quad 10^{-7} \leq v_s \leq 10^{-5}; \quad 0 \leq E_s \leq 100 \]

(19a-c)

Analysis of these large numbers of optimal sections so obtained for all the three type of canal sections [1], indicated that the linear dimensions are proportional to the length scale \( L \) (m) given by

\[ L = \lambda \left( e_s + 8v_s \right)^{0.04} \]

(20)

Further the analysis [1] resulted in the following generalized
equations for the optimal dimensions and the corresponding water loss for all the three canal sections:

\[ m' = k_{ms} \left[1 + k_{me} \left(\frac{E}{k}\right)\right]^{1 - \gamma} ; \quad b' = k_{bs} L \left[1 + k_{be} \left(\frac{E}{k}\right)^{1 - \gamma} \right]^{1 - \gamma} \tag{21a-b} \]

\[ y' = k_{ys} [1 + k_{ye} \left(\frac{E}{k}\right)^{1 - \gamma}]^{1 - \gamma} ; \quad q' = k_{qs} KL \left[1 + k_{qe} \left(\frac{E}{k}\right)^{1 - \gamma} \right]^{1 - \gamma} \tag{21c-d} \]

where * indicates optimality; \( k_{\beta} \) = coefficients; and \( r_{I} \) and \( s_{I} \) = exponents. The first subscript \( m, b, y, \) and \( q \) denote side slope, bed width, normal depth, and water loss respectively and the second subscript \( s, e \) denote seepage, and evaporation loss respectively. Table 1a lists the section shape coefficients. The optimal section properties with Manning’s equation are tabulated in Table 1b, in which case

\[ L = \lambda = \left(\frac{Qn}{\sqrt{S_0}}\right)^{3/8} \tag{22} \]

where \( n \) = Manning’s roughness coefficient.

Table 1a. Properties of Optimal Canal Sections (General Equation)

<table>
<thead>
<tr>
<th>Entity</th>
<th>Coefficients or Exponents (1)</th>
<th>Section Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(2)</td>
<td>Triangular</td>
</tr>
<tr>
<td>Side Slope</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k_{ms} )</td>
<td>1.2466</td>
<td>0.5984</td>
</tr>
<tr>
<td>( k_{me} )</td>
<td>0.4850</td>
<td>0.3106</td>
</tr>
<tr>
<td>( r_{m} )</td>
<td>0.9020</td>
<td>1.0937</td>
</tr>
<tr>
<td>( s_{m} )</td>
<td>1.1732</td>
<td>5.0000</td>
</tr>
<tr>
<td>Bed Width</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k_{bs} )</td>
<td>0.7986</td>
<td>0.5446</td>
</tr>
<tr>
<td>( k_{be} )</td>
<td>1.0717</td>
<td>0.1561</td>
</tr>
<tr>
<td>( r_{b} )</td>
<td>0.9849</td>
<td>2.2409</td>
</tr>
<tr>
<td>( s_{b} )</td>
<td>0.3798</td>
<td>0.1616</td>
</tr>
<tr>
<td>Normal Depth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k_{ys} )</td>
<td>0.4518</td>
<td>0.3178</td>
</tr>
<tr>
<td>( k_{ye} )</td>
<td>0.3895</td>
<td>0.5198</td>
</tr>
<tr>
<td>( r_{y} )</td>
<td>0.9286</td>
<td>0.8994</td>
</tr>
<tr>
<td>( s_{y} )</td>
<td>0.7114</td>
<td>0.6630</td>
</tr>
<tr>
<td>Water Loss</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k_{qs} )</td>
<td>2.0015</td>
<td>2.0399</td>
</tr>
<tr>
<td>( k_{qe} )</td>
<td>0.9084</td>
<td>0.5707</td>
</tr>
<tr>
<td>( r_{q} )</td>
<td>1.0126</td>
<td>0.9433</td>
</tr>
<tr>
<td>( s_{q} )</td>
<td>0.6241</td>
<td>0.6376</td>
</tr>
</tbody>
</table>

Fig. 2 plots the behaviour of the design equations (21). In Fig. 2 variations of side slope and the dimensionless bed width, normal depth, and water loss with \( E/k \) were plotted for triangular, rectangular, and trapezoidal canals. Fig. 2 shows that side slope and bed width of the optimal section decrease and normal depth of the optimal section increases with increase in \( E/k \). A perusal of Fig. 2(b) for water loss reveals that the optimal triangular section permits less water loss than the optimal rectangular section in the range \( 0 \leq E \leq 0.14 \), otherwise the rectangular section is more efficient than the triangular section. The optimal trapezoidal section loses the least water amongst the three optimal sections for the full range of \( E \). For \( E \geq 2 \), the losses from the optimal rectangular section and from the trapezoidal section are nearly the same. For \( E < 0.1 \), the water loss and the canal dimensions are less sensitive to \( E/k \) and more sensitive otherwise. At \( E = 0 \), (21) gives
the optimal dimensions and water loss for a minimum seepage loss canal section [14].

For a given set of data, the use of (20) and (21a-c), along with Table 1 results in the optimal canal section. Section shapes coefficients for water loss along with (21d) gives the minimum water loss from the optimal section. Alternatively the losses from the optimal section can be obtained using (8-10), once the section dimensions are fixed. For the designed section the average flow velocity \( V \) (m/s) can be obtained by the continuity equation

\[
V = Q / A
\]  
(23)

Table 1b. Properties of Optimal Canal Sections (Manning equation)

<table>
<thead>
<tr>
<th>Entity (1)</th>
<th>Coefficients or Exponents (2)</th>
<th>Section Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Triangular (3)</td>
<td>Rectangular (4)</td>
</tr>
<tr>
<td>Side Slope</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k_{ms} )</td>
<td>1.2407</td>
<td>0.5981</td>
</tr>
<tr>
<td>( k_{me} )</td>
<td>0.4577</td>
<td>0.2983</td>
</tr>
<tr>
<td>( r_b )</td>
<td>0.8865</td>
<td>1.0875</td>
</tr>
<tr>
<td>( s_b )</td>
<td>1.2066</td>
<td>5.0000</td>
</tr>
<tr>
<td>Bed Width</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k_{bs} )</td>
<td>2.0545</td>
<td>1.3959</td>
</tr>
<tr>
<td>( k_{be} )</td>
<td>1.0747</td>
<td>0.1422</td>
</tr>
<tr>
<td>( r_b )</td>
<td>0.9841</td>
<td>2.2628</td>
</tr>
<tr>
<td>( s_b )</td>
<td>0.3718</td>
<td>0.1567</td>
</tr>
<tr>
<td>Normal Depth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k_{ns} )</td>
<td>1.1676</td>
<td>0.8211</td>
</tr>
<tr>
<td>( k_{ne} )</td>
<td>0.3643</td>
<td>0.4961</td>
</tr>
<tr>
<td>( r_y )</td>
<td>0.9184</td>
<td>0.8943</td>
</tr>
<tr>
<td>( s_y )</td>
<td>0.7371</td>
<td>0.6760</td>
</tr>
<tr>
<td>Water Loss</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( k_{qs} )</td>
<td>5.1614</td>
<td>5.2599</td>
</tr>
<tr>
<td>( k_{qe} )</td>
<td>0.8882</td>
<td>0.5612</td>
</tr>
<tr>
<td>( r_q )</td>
<td>1.0104</td>
<td>0.9432</td>
</tr>
<tr>
<td>( s_q )</td>
<td>0.6346</td>
<td>0.6464</td>
</tr>
</tbody>
</table>

This average velocity should be greater than the non-silting velocity but less than the limiting velocity \( V_L \). The limiting velocity depends on the lining material as given in Table 2 [11]. If \( V \) is greater than \( V_L \), a superior lining material should be selected.

7. Design Example

Design a minimum water loss concrete lined rectangular canal section for carrying a discharge of 10 m³/s on a longitudinal slope of 0.001. The canal lining has \( \varepsilon = 1 \) mm. Assume canal lining as cracked; and having \( k = 10^{-6} \) m/s. The maximum evaporation loss \( E \) was estimated as 2.5x10⁶ m²/s. The water temperature is 20 °C at which \( V = 1.1x10^{-6} \) m²/s.

Solution

Adopting \( g = 9.79 \) m/s² and using (12) \( \lambda = 6.336 \) m. Using (13) \( \tau = 1.578x10^{-4}; \nu = 6.970x10^{-7}; \text{ and } E = 2.5 \). Using (20) \( L = 4.471 \) m. For a rectangular section Table 1a gives the bed width parameters as: \( k_{bs} = 0.7986; k_{be} = 1.0717; r_b = 0.985; \text{ and } s_b = 0.3798 \). Using these parameters and (21b), \( b^* = 2.186 \) m. Similarly the normal depth parameters are \( k_{ns} = 0.3178; k_{ne} = 0.5198; r_y = 0.8994; \text{ and } s_y = 0.6630 \). Using these parameters and (21c), \( y_n^* = 2.386 \) m. Adopting \( b = 2.19 \) m and \( y_n = 2.39 \) m, \( A = 5.234 \) m²; and using (23) \( V = 1.91 \) m/s, which is within permissible limit (Table 2). Using (9) the seepage loss \( q_s = 1.032x10^{-5} \) m²/s; and the evaporation loss \( q_e = 5.475x10^{-6} \) m²/s. Summing up these losses, loss \( q_w = 1.580x10^{-5} \) m²/s; while the direct use of (21d) gives \( q_w = 1.575x10^{-5} \) m²/s.

Table 2. Limiting Velocities

<table>
<thead>
<tr>
<th>Lining Material</th>
<th>Limiting Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boulder</td>
<td>1.0-1.5</td>
</tr>
<tr>
<td>Brunt Clay Tile</td>
<td>1.5-2.0</td>
</tr>
<tr>
<td>Concrete Tile</td>
<td>2.0-2.5</td>
</tr>
<tr>
<td>Concrete</td>
<td>2.5-3.0</td>
</tr>
</tbody>
</table>

Similarly, using the same data for design of a triangular canal yields: \( m = 0.52; \text{ and } y_n^* = 3.203 \) m. Thus, \( A = 5.335 \) m²; and \( V = 1.874 \) m/s. Using (8) \( q_s = 1.052x10^{-5} \) m²/s; \( q_e = 8.328x10^{-6} \) m²/s; and \( q_w = 1.885x10^{-5} \) m²/s, but \( q_w = 1.884x10^{-5} \) m²/s from (21d). Thus in comparison to rectangular section the triangular section is less efficient.

8. Conclusions

Explicit design equations and optimal section shape coefficients for triangular, rectangular and trapezoidal sections have been obtained to facilitate design of minimum water loss canal sections.

Notation

- \( A \) flow area [m²];
- \( b \) bed width of canal [m];
- \( E \) evaporation discharge per unit free surface area [m/s];
- \( e_d \) saturation vapour pressure at dew point temperature [Pa];
- \( e_v \) saturation vapour pressure at water surface temperature [Pa];
- \( F_s \) seepage function [dimensionless];
- \( f_w \) wind function [m²/Pa];
- \( g \) gravitational acceleration [m/s²];
- \( L \) length scale [m];
- \( k \) coefficient of permeability [m/s];
- \( k_{fs} \) section shape coefficients [dimensionless];
- \( m \) side slope [dimensionless];
- \( n \) Manning’s roughness coefficient [dimensionless];
- \( Q \) discharge [m³/s];
- \( q_e \) evaporation loss per unit length of canal [m²/s];
- \( q_s \) seepage loss per unit length of canal [m²/s];
\( q_w \) total water loss per unit length of canal \([\text{m}^2/\text{s}]\);
\( R \) hydraulic radius \([\text{m}]\);
\( R_h \) relative humidity \([\text{dimensionless}]\);
\( r_f \) exponents \([\text{dimensionless}]\);
\( S_0 \) bed slope \([\text{dimensionless}]\);
\( s_f \) exponents \([\text{dimensionless}]\);
\( T \) width of free surface \([\text{m}]\);
\( u_2 \) wind velocity at 2 m above water surface \([\text{m/s}]\);
\( V \) average velocity \([\text{m/s}]\);
\( V_L \) limiting velocity \([\text{m/s}]\);
\( y_n \) normal depth \([\text{m}]\);
\( \varepsilon \) roughness height \([\text{m}]\);
\( \lambda \) length scale \([\text{m}]\);
\( \nu \) kinematic viscosity \([\text{m}^2/\text{s}]\);
\( \phi \) equality constraint \([\text{dimensionless}]\);
\( \rho \) penalty parameter \([\text{dimensionless}]\);
\( \psi \) augmented function \([\text{dimensionless}]\);
\( \theta_a \) mean air temperature \([\degree \text{C}]\); and
\( \theta_w \) water surface temperature \([\degree \text{C}]\).

**Subscript**
- \( b \) bed width;
- \( e \) evaporation;
- \( m \) side slope;
- \( q \) water loss;
- \( s \) seepage loss;
- \( y \) normal depth; and
- \( * \) non-dimensional.

**Superscript**
- \( * \) optimal.

**References**