Identification of Dynamic Parameters of an Industrial Manipulator

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ABSTRACT

This paper discusses the identification of dynamic parameters of an industrial robot KUKA KR5 with all revolute joints and serial architecture. For this, the simplified model of the robot was taken into account by considering only those joints which normally operates orthogonal to the gravity vector. The first approach used for finding the dynamic parameters of this simplified model was by formulating the dynamic model using Euler-Lagrangian (EL) method symbolically. Then the equations of motion obtained were linearized and expressed in terms of the base parameters. The numerical values of base parameters were obtained by linear regression technique applied to the points along a given planar trajectory. Robot sensory interface (RSI) of KUKA KRC2 controller was used to obtain the joint torque and position values. The dynamic parameters obtained were verified by comparing the mass moment obtained from another approach of curve fitting to the joint torque values.

1. INTRODUCTION

Serial architecture with model-based control is extensively used for industrial robots to perform tasks such as pick and place, painting, arc welding, assembly of components, etc. As these operations require complex motion to be followed, accurate dynamic model of the rigid body manipulator is required for achieving better motion control, performance of the manipulator, torque prediction, etc. By dynamic model we mean first order mass moment (mass multiplied by position of center of mass) and the second order mass moment called six inertial parameters. The accuracy of the dynamic model of a robot depends upon its geometric parameters and the dynamic parameters. Precise geometric parameters can be obtained through kinematic calibration whereas there exist a number of methods for identifying the dynamic parameters such as weighing the individual links of the manipulator, using CAD model with material properties of each link, through experiments based upon the manipulator input and output relation, adaptive control algorithm, etc.

In general, a standard robot identification procedure consists of modeling, experiment design, data acquisition, signal processing, parameter estimation, and model validation [1]. The mathematical model of a robotic manipulator can be obtained from analytical mechanics like using Lagrangian equation, Newton-Euler equations, etc., they are discussed in [2, 3]. The mathematical model obtained contains position, velocity and acceleration of the links which are used to determine the minimal inertial parameters of the robot [4, 5]. The dynamic parameters for a robot manipulator are typically classified as unidentifiable, identifiable in linear combination, and uniquely identifiable. The maximum number of identifiable dynamic parameters, are known as base parameters [6].

One of the approaches in off-line identification is by physical experiments where the robot is disassembled and the inertial parameters are obtained experimentally. Some of the techniques for measuring the inertia tensor are discussed in [7]. Atkeson et al. [8] proposed the estimation of inertial parameters. Gautier et al. [5] proposed a dynamic identification method by using only the torque data. The torque estimates are used in gravity compensation as discussed in [9]. The joint torque values are also required for force reflecting devices like exoskeleton, haptic devices [10], tele-operation, master-slave manipulator etc, in which the weight of the driving links should be compensated.

In this paper, the dynamic parameters are identified using experimental data and validated by comparing the mass moments obtained using two methods illustrated in this paper in Sections 2.3 and 2.4. The
kinematic parameters, i.e., the DH parameters of an industrial robot KUKA KR5 were first identified using circle point method in [11] and are used here as geometric description. The dynamic modeling is done using Euler Lagrangian method and a simplified model of KUKA KR5 was considered by taking only joints 2, 3 and 5 into account. The base parameters were then identified for the joints considered by grouping of inertial parameters, which gives the regressor form of the dynamic model. The joints 2, 3 and 5 were provided with a planar trajectory and the torques and position data were recorded from the Robot Sensory Interface (RSI). The torque values obtained was used to find the mass moment by fitting the expression using Fourier fit. The results are then compared for a general trajectory provided to these joints with the identified torque values and obtained torque data.

This paper is divided into four sections. In Section 2, the basic formulation for the dynamic model and the base parameters are presented. In Section 3, the methodology for the data acquisition, extraction of the dynamic parameters are presented. Validation and results with concluding remarks are presented in Section 4.

2. FORMULATION

In this section the the description of the kinematic and dynamic parameters which is essential for the dynamic model formulation identification are discussed.

2.1. Kinematic and dynamic parameters of robot

The most widely used kinematic notation for the geometric modeling is Denavit-Hartenberg (DH) notation. Figure 1 shows the DH parameters and their definitions in tabular form, as given in Table 1. The travel from the base frame to the end-effector frame is achieved by moving across two consecutive frames placed at the joints. They are indicated in Figure 1.

![Figure 1. Spatial link with highlighted DH parameters.](image)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_i )</td>
<td>Joint offset</td>
<td>( X_i \rightarrow X_{i+1} ) @ ( Z_i )</td>
</tr>
<tr>
<td>( \theta_i )</td>
<td>Joint Angle</td>
<td>( X_i \rightarrow X_{i+1} ) rotation, ccw</td>
</tr>
<tr>
<td>( a_i )</td>
<td>Link Length</td>
<td>( Z_i \rightarrow Z_{i+1} ) @ ( X_{i+1} )</td>
</tr>
<tr>
<td>( \alpha_i )</td>
<td>Twist Angle</td>
<td>( Z_i \rightarrow Z_{i+1} ) rotation, ccw</td>
</tr>
</tbody>
</table>

*In the table read symbol \( \rightarrow \) as “to”, \( \perp \) as perpendicular”, \( @ \) as “about or along”

Using the definition of Table 1 the kinematic parameters identified for the KUKA KR5 of Figure 2 in [11]. They are given in Table 2.
Table 2. DH parameters of the KUKA KR5

<table>
<thead>
<tr>
<th>Joint No.</th>
<th>Joint Offset ((b)) (mm)</th>
<th>Joint Angle (\theta) (variable)</th>
<th>Link Length ((a)) (mm)</th>
<th>Twist Angle ((\alpha)) (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400.011</td>
<td>(\theta_1)</td>
<td>179.992</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>(\theta_2)</td>
<td>599.998</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.002</td>
<td>(\theta_3)</td>
<td>119.991</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>620.042</td>
<td>(\theta_4)</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>5</td>
<td>0.001</td>
<td>(\theta_5)</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>(\theta_6)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The forward kinematics, i.e., to get the value of the pose of the end-effector of the robot with respect to the robot base link frame, from the joint variables values and geometric parameters, can be given as,

\[
T = \prod_{i=0}^{6} (T_{i} T_{i+1} T_{i+2} T_{i+3})
\]  

(1)

Note now Figure 3, which shows a general link \(i\), with its mass \(m_i\) and center of mass (COM) located at \(C_i\). Link length is shown using the DH convention as \(a_i\), i.e., the distance between \(Z_i\) and \(Z_{i+1}\) along \(X_i\). The moment of inertia \(I_i\) are usually taken about the center of mass of each link which results in the expression for the joint torque being non-linear. The inertia tensor about link frames is given in Equation (2)

\[
I_i^O = I_i^C - mr (\tilde{r} \times \tilde{r})
\]  

(2)

where \(I_i^O\) and \(I_i^C\) are the inertia tensors of link \(i\) about its link-fixed origin and centre of mass respectively, and \(\tilde{r}\) is the skew symmetric matrix associated to \(r\) which is defined as

\[
\tilde{r} = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}
\]  

(3)

The expression in Equation (2) in different reference frame, say, the fixed frame, tensor \([I_i^C]_{i+1}\) can be obtained by multiplying it with the rotation matrix \(Q_{i+1}\) as in Equation (4)

\[
[I_i^C]_{i+1} = Q_i [I_i^C] Q_i^T
\]  

(4)

The dynamic parameters, i.e., the inertial parameters of a serial manipulator which appears in the expression of joint torques, are the six moments of inertia terms, three mass moments terms and one mass. With moments of inertia terms expressed about link frames, these constitute the 10\(n\) inertial
parameters of an \( n \)-degrees of freedom serial manipulator. These are also known as the standard inertial parameters (SIP) that are represented as vector \( \mathbf{x}_i \) is then defined as
\[
\mathbf{x}_i = \begin{bmatrix}
I_{xx,i} & I_{xy,i} & I_{xz,i} & I_{yy,i} & I_{yz,i} & I_{zz,i} & m_{x,i} & m_{y,i} & m_{z,i}
\end{bmatrix}^T
\] (5)

2.2. Dynamic Formulation

The Lagrangian formulation for the dynamic model is given by:
\[
\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \tau_i
\] (6)
where \( L \) is the Lagrangian function, i.e., the difference between total kinetic energy (T) and the potential energy (U), \( q_i \) is the \( i \)-th generalized coordinate of the system at hand, \( \dot{q}_i \) is rate of change of the \( i \)-th generalized coordinate and \( \tau_i \) is the generalized force. Equation (6), for \( i = 1, 2, ..., n \),
\[
\mathbf{I}(q) \ddot{q} + \mathbf{h}(q, \dot{q}) + \gamma(q) = \tau
\] (7)
where \( \mathbf{I} \) is the generalized inertia matrix, \( \mathbf{h} = [h_1, h_2, ..., h_n]^T \) is the \( n \)-dimensional vector of centrifugal and coriolis forces, \( \gamma = [\gamma_1, \gamma_2, ..., \gamma_n]^T \) is the \( n \)-dimensional vector of gravitational forces and \( \tau = [\tau_1, \tau_2, ..., \tau_n]^T \) is the \( n \)-dimensional vector of generalized forces. The expression for the \( \mathbf{I}, \mathbf{h} \) and \( \gamma \) are obtained symbolically in MATLAB using the expression given in book [3].

2.3. Regressor Formulation

As explained in section 2.1, the joint torque expression is not linear in the inertial parameters as used in EL formulation. However when converted in terms of standard inertial parameters the expression is linear, which is exploited for the dynamic parameter identification. This is achieved by extracting the coefficients of the standard inertial parameters from the symbolic torque expression to give the linear form as shown in Equation (8).
\[
\tau = \mathbf{Y}(q, \dot{q}, \ddot{q}) \chi = \mathbf{Y}\chi
\] (8)
where \( \chi \) is \( 10n \) dimensional vector containing standard inertial parameters (SIP) of the manipulator, \( \mathbf{Y} \) is the \( n \times 10n \) matrix containing coefficients corresponding to SIP. The coefficients were collected using \([Q,R] = \text{quorem}(A,B,\text{var})\) function in MATLAB where \( A \) and \( B \) are the symbolic matrices and \( \text{var} \) corresponds to variable. In place of \( A \) the symbolic expression for the torque obtained from Equation (7) was placed and for \( B \) and \( \text{var} \) following expressions are used, where \( i \) is the number of links and the previous links were stacked in a row to result in \( i \times 10 \) matrix as
\[
\mathbf{B} = \begin{bmatrix}
I_{xx,i} & I_{xy,i} & I_{xz,i} & I_{yy,i} & I_{yz,i} & I_{zz,i} & m_{x,i} & m_{y,i} & m_{z,i}
\end{bmatrix}_{i=10}
\]
\[
\text{var} = \begin{bmatrix}
I_{xx,i} & I_{xy,i} & I_{xz,i} & I_{yy,i} & I_{yz,i} & I_{zz,i} & r_{x,i} & r_{y,i} & r_{z,i}
\end{bmatrix}_{i=10}
\] (9)
This gives the quotient on division of a symbolic polynomial expression by variables listed. The fact that the coefficient of moment of inertia terms about link origin and that about the centre of mass are same was used to collect the coefficients of inertia tensor terms. The coefficients of first order moment terms were obtained by first removing the terms containing products of position of center of mass, which are those appearing in the Equation (2). This was done by first taking the product of these terms with the coefficients of corresponding standard inertia terms and then subtracting these products from the torque expressions. After removal of the product terms, \text{quorem} function was used to collect the remaining coefficients. Similarly for the coefficient of mass term of standard inertial parameter, first all the terms of prior inertial parameters were removed and then quorem function was applied with mass as variable. There can be some standard inertial parameters on which the dynamics of the manipulator does not depend. These redundant parameters were found once the linear form of the dynamic equations was obtained, as shown in Equation (8). The columns of matrix \( \mathbf{Y} \) which have zeros correspond to the redundant parameters in matrix \( \chi \). For example, if the \( n^{th} \) column of matrix \( \mathbf{Y} \) is the zero column then \( n^{th} \) parameter in the matrix \( \chi \) is the redundant parameter. These redundant standard inertial parameters and
the corresponding zero columns in the coefficient matrix were removed by taking the steps discussed in this section.

Note that the matrix $Y$ even after removing the zero columns may not be of full column rank and in that case not all the non-redundant standard inertial parameters affect dynamics independently. For example, if $\beta^{th}$ column of matrix $Y$ is linearly dependent in the following form:

$$ Y_i = t_{i1}Y_1 + t_{i2}Y_2 + \ldots + t_{ip}Y_p \tag{10} $$

then corresponding inertial parameter $\chi_i$ can be eliminated and regrouped with other inertial parameters with the coefficients as in Equation (10). Thus the set $\chi$ can be reduced to minimum $b$ number of parameters, where $b$ is the column rank of matrix $Y$. This reduced set of regrouped parameters $\chi_i$ is the base parameter set for the manipulator at hand. The corresponding $Y$ matrix after eliminating all zero or linearly dependent columns gives the regressor matrix $Y^\#$ and regressor form of dynamic equations as

$$ \tau = Y^\# \chi_b \tag{11} $$

where $Y^\#$ is the $nxb$ Regressor matrix and $\chi_b$ is the $bx1$ matrix of base parameters which affects the dynamics of the system.

The formulation of the regressor matrix and base parameters was done through numerical method employing QR decomposition. The matrix $Y$ after removing zero columns, was evaluated at 500 time samples with known cycloidal joint trajectories shown below:

$$ \theta = \theta(0) + \frac{\theta(T)-\theta(0)}{T} \left[ t - T \sin \left( \frac{2\pi}{T} \right) \right] \tag{12} $$

where $\theta(0)=0$, $\theta(T)=\pi/3$ and $T = 5$sec. The joint rate and acceleration were found by differentiating the expression Equation (12). The kinematic parameters used are shown in Table 2. The regressor matrix is evaluated 500 times and are stacked one below the other to form the observation matrix $B$ of dimension $500\times bx$. The QR decomposition of this observation matrix $B$ gave an orthogonal matrix $Q$ and an upper triangular matrix $R$. The diagonal element of $R$ having zero element, then the corresponding column in $B$ is linearly dependent on the other columns [12]. The dependent columns are placed in matrix $B$ with subscript $d$ as $B_d$. Thus by comparing diagonal elements of $R$ with numerical zero one can rearrange terms of $B$ to form $B=\left[ B_n \ B_d \right]$.

$$ \left[ B_n \quad B_d \right] = \left[ Q_1 \quad Q_2 \right] \left[ \begin{array}{cc} R_1 & 0 \\ 0 & 0 \end{array} \right] = \left[ Q_1R_1 \quad Q_2R_2 \right] \tag{14} $$

$B_d$ in terms matrix $B_n$ can be written as,

$$ B_d = B_n \beta \tag{13} $$

Similarly the regressor matrix can be written after above decomposition as $Y=[Y_1 \ Y_2]$ and $[K_1 \ K_2]^T$ as,

$$ \tau = [Y_1 \ Y_2] \left[ \begin{array}{c} K_1 \\ K_2 \end{array} \right] \tag{15} $$

$$ \tau = Y_1K_b \quad \text{where,} \quad K_b=K_1+\beta K_2 \tag{16} $$

where $R_1$ is $bxh$ matrix where $b$ is also the rank of observation matrix and $R_2$ is $bx(p-b)$ matrix, $b$ is the number of independent columns in $B_1$ and $p$ is the total number columns in $B_2$. Hence,

$$ \beta = (R_1)^{+}R_2 \tag{17} $$

Once the matrix $\beta$ was found, the final regressor form of the dynamic equations can be written using the Equations (14) and (15), where $Y_1$ is the regressor matrix and $K_b$ is the matrix of base parameters as in Equation (15).

2.4 Identification of mass-moments

The second approach to find the mass moments of the robot is explained in this section. From Figure 3 the actuator torque for the $i^{th}$ revolute joint, $\tau_i$ was obtained by projecting the driving moments onto their corresponding joint axes. For static moment due to gravity only, i.e., $[n_{i,1}]$, is given by,

$$ \tau_i = [e_i]^{\dagger} [n_{i,1}] \quad : \text{for a revolute joint} \tag{18} $$
where \([\mathbf{n}_{i,i+1}]\) is the moment on \(#i\) by \(#(i+1)\) which is represented in Frame \(i\), whereas \([\mathbf{e}_{i}]\) is the unit vector parallel to the axis of rotation of the \(i^{th}\) joint represented in the \(i^{th}\) frame where it is simply \(\mathbf{e}_{i} = [0 \ 0 \ 1]^T\) [3]. Note in static equilibrium

\[
[\mathbf{n}_{i,i+1}] = m_i \mathbf{d} \times \mathbf{g}
\]

Expanding the vector representation in Frame \(i\), Equation (18), can be rewritten as

\[
\tau_i = m_i \mathbf{d} \cdot \mathbf{g} \sin(\theta_i \pm \phi_i)
\]

It is possible out here that the expression of \(\tau_i\) in Equation (18) can be recursively obtained from the outer link to the inner link obtained as

\[
\tau_{i+1} = \tau_i + m_i \left[ \mathbf{d} \times \mathbf{g} \right]_{i+1} \mathbf{e}_{i+1}
\]

Equation (20) and (21) are the expressions which will be used for the identification of the mass moments. It was observed that the variation of the torques were sinusoidal in nature as the inputs to joints were sinusoidally moved. Fourier approximation was used to best fit the curves and find the mass moments as in [13], i.e.,

\[
\tau_i = a_0 + \sum_{n=1}^{\infty} a_n \sin(n\phi) + b_n \cos(n\phi)
\]

where \(\tau\) is the torque at the \(i^{th}\) joint. Moreover \(a_n, a_n, b_n\) are constants, called the coefficients of the series, \(n\) is an integer, and \(\phi\) is the angle swept by the links.

3. IDENTIFICATION

This section explains the methods used to find the base parameters of an industrial robot KUKA KR5. The dynamic parameter namely base parameters and the mass moment will be identified for the links taken into account shown in this section.

3.1. Data Acquisition, processing and filtering

For dynamic parameter identification, the joint position, velocity, acceleration and torque data is required at each joint. The Robot Sensory Interface (RSI) [14] gives only the joint position (degrees) and torque (Newton-meter) data each 12 miliseconds interval. The joint variation and corresponding torque values are plotted for 500 time step for joints is shown in Figure 3:

![Figure 3](image-url)

**Figure 3.** Joint variation and the corresponding torque values of respective joints obtained from RSI

The joint velocity \(\dot{\theta}\) was obtained by differentiating the joint position data \(\theta\) and the acceleration \(\ddot{\theta}\) is obtained by subsequent differentiation of the velocity data as shown in Figure 5. For removing the noise in velocity and acceleration, butterworth low pass filter of second order was applied using MATLAB. The filtered velocity and acceleration data alongwith the joint position and torque were further used for the dynamic parameter identification.
3.2. Extraction of Dynamic parameters

The simplified model of the robot was considered by considering only joints 2, 3 and 5. Links 1, 4 and 6 were not actuated which restricts the robot to move in a plane parallel to the gravity vector. Links 2, 3 and 5 are orthogonal to the direction of the gravity which were utilised in most of the industrial task and affects the dynamical model most. Here Axes 1 and 6 are along the direction of the gravity vector and are not allowed to move. Hence, the inertial parameters listed in Equation (9) will have no effect on the dynamical model. For the 3 link planar mechanism formed by giving motions to joints 2, 3 and 5 of KUKA KR5 manipulator, the total number of standard inertial parameters (SIP) is 30, as discussed in section 2.1. For brevity the links of the industrial robot KUKA KR5 and the corresponding links of the 3 link equivalent planar system are numbered 1, 2 and 3, as listed in Table 3 and shown in Figure 6. The DH parameters of the resulting manipulator obtained using the identified kinematic parameters are shown in Table 3.

Table 3. DH parameters of three link planar KUKA KR5

<table>
<thead>
<tr>
<th>Link no. of planar robot</th>
<th>Link nos. of KUKA KR5</th>
<th>Joint Offset (b) (mm)</th>
<th>Joint Angle ( \theta ) (variable)</th>
<th>Link Length (a) (mm)</th>
<th>Twist Angle (( \alpha )) (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>#2</td>
<td>0</td>
<td>( \theta_1 ) 0.6</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>(#3)+(#4)</td>
<td>0</td>
<td>( \theta_2 ) 0.631</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>(#5)+(#6)</td>
<td>0</td>
<td>( \theta_3 ) 0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

The Regressor formulation, as explained in section 2.3, was done using the DH parameters given in Table 3. Out of the 30 standard inertial parameters (SIP), 18 were found to be redundant. The remaining 12 parameters affecting the manipulator dynamics are listed below:

\[
[ I_{z,z}, m_1 r_{z1}, m_1 r_{z1}, I_{z,z}, m_2 r_{z2}, m_2 r_{z2}, I_{z,z}, m_3 r_{z3}, m_3 r_{z3}, m_3 ]
\]  

Further these were regrouped to give the final 9 base parameters listed in Table 5. For dynamic parameter identification, joint position, velocity, acceleration and torque data at 500 time samples were used, i.e., \( \theta, \dot{\theta}, \ddot{\theta} \) and \( \tau \) are input to the dynamic model obtained in Equation (10). The numerical matrices formed by
substituting the joint motion data in the Regressor matrix were stacked one below the other to form the observation matrix $Y$ of dimension $1500 \times b$. The joint torque data was similarly stacked one below the other to form the torque matrix of dimension $1500 \times 1$. The estimated base parameter values were obtained through the least squares method and are given as

$$K_s = Y^+ \tau$$

where $K_s$ is the set of estimated base parameters and $Y^+$ is the pseudo inverse of matrix $Y$.

3.3. Results and validation

The nine base parameters and their identified values are shown in the Table 4.

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Base Parameters</th>
<th>Identified Values</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$I_{xx,1} - 0.36m_i - 0.36m_2 - 0.36m_3$</td>
<td>-178.294</td>
<td>Kg-m$^2$</td>
</tr>
<tr>
<td>2</td>
<td>$0.6m_1 + 0.6m_2 + 0.6m_3 + m d_{s,1}$</td>
<td>38.3942</td>
<td>Kg-m</td>
</tr>
<tr>
<td>3</td>
<td>$m d_{s,1}$</td>
<td>0.0064</td>
<td>Kg-m</td>
</tr>
<tr>
<td>4</td>
<td>$I_{zz,1} - 0.39879m_i - 0.39879m_2$</td>
<td>3.9082</td>
<td>Kg-m$^2$</td>
</tr>
<tr>
<td>5</td>
<td>$0.6315m_1 + 0.6315m_2 + m d_{s,3}$</td>
<td>10.6798</td>
<td>Kg-m</td>
</tr>
<tr>
<td>6</td>
<td>$m d_{s,2}$</td>
<td>7.2487</td>
<td>Kg-m</td>
</tr>
<tr>
<td>7</td>
<td>$I_{zz,3}$</td>
<td>14.9955</td>
<td>Kg-m$^2$</td>
</tr>
<tr>
<td>8</td>
<td>$m d_{s,3}$</td>
<td>1.1736</td>
<td>Kg-m</td>
</tr>
<tr>
<td>9</td>
<td>$m d_{s,3}$</td>
<td>-0.4988</td>
<td>Kg-m</td>
</tr>
</tbody>
</table>

The identified values of the dynamic parameters were used to obtain the joint torques for a given motion to joints 2, 3 and 5 of the KUKA KR5 manipulator. The torques obtained were compared with those obtained from the Robot Sensory Interface (RSI). Also the predicted error torques are plotted with the marker shown after every 25 data points which are shown in Figure 7.

(a) Measured torque from RSI Vs. estimated torques from dynamic model of joint 1

(b) Torque prediction error between estimated and measured values for joint 1
The error plot between the joint torques obtained from the RSI and the identified dynamic model shows that there are good match between them and hence the identified dynamic model is also right. The identified dynamic parameters are also compared with the mass moments values obtained through the gravity compensation method and are shown in Table 5.

**Table 5.** Comparison of identified mass moment values obtained through the two methods

<table>
<thead>
<tr>
<th>Dynamic Parameters</th>
<th>Identified values through Inverse Dynamic Least Squares method</th>
<th>Identified values through Gravity Compensation method [13]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.6m_1 + 0.6m_2 + 0.6m_3 + m_1 d_{y1}$</td>
<td>38.3942</td>
<td>38.297</td>
</tr>
<tr>
<td>$m_1 d_{y1}$</td>
<td>0.0064</td>
<td>0</td>
</tr>
<tr>
<td>$0.6315m_1 + 0.6315m_3 + m_1 d_{y2}$</td>
<td>10.6798</td>
<td>8.612</td>
</tr>
<tr>
<td>$m_2 d_{y2}$</td>
<td>7.2487</td>
<td>9.058</td>
</tr>
<tr>
<td>$m_3 d_{y3}$</td>
<td>1.1736</td>
<td>1.169</td>
</tr>
<tr>
<td>$m_3 d_{y3}$</td>
<td>-0.4988</td>
<td>-0.499</td>
</tr>
</tbody>
</table>

### 4. CONCLUSIONS

This paper presents the identification of dynamic parameters of a simplified model of a KUKA KR5 robot manipulator. It consists of three joints, namely, 2, 3, and 5. The dynamic equations were written in linear...
form in terms of the base parameters with the help of symbolic toolbox in MATLAB. The parameters identified using the regressor formulation matched quite closely with those obtained using gravity compensation technique. While the former methodology is quite generic, the later has limitations, i.e., unless the robot moves against the gravity it will not be possible to identify the associated parameters.

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