Extraction of Fractal Dimension for Iris Texture

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Abstract

Fractal dimension has not been an effective feature for authentication using the iris. An attempt is made in this paper to explore its effectiveness on iris texture by deriving it from the features obtained using the Information Set based methods. In addition to this, a surface finish measurement tool called Topothesy is also employed for determining the fractal dimension. The well-known Box-Counting method is discussed in brief to facilitate comparison with the new approaches. All the approaches have been tested on iris images from CASIA iris v3 database. The results of comparison with the tested approaches indicate that information set based method and topothesy based method perform better than the traditional methods. The results demonstrate the feasibility of using fractal features to quantify biometric traits like iris feature as well as the superiority of the newly proposed Information set based methods over the traditional approaches

Keywords: Fractals, Topothesy, Differential Box-Counting,Feature extraction, Fuzzy sets, Biometric classification

1 Introduction

The iris is one of the best biometrics for personal authentication [1]. It contains textural information which has been extracted using different approaches based on Gabor Filter, wavelets etc. Fractals are ways of characterizing geometries found in nature using mathematical terms and were first introduced by Benoit Mandelbrot. Fractals exhibit self-similarity i.e. the whole structure is approximately similar in shape to its constituent parts. Given the structure of iris texture, a fractal based approach holds considerable promise. Traditional approaches like Box-Counting [2], [3] have already been implemented in the coarse classification of the iris. Here in this paper, modes of extraction of fractal dimension using a fuzzy approach [4], and topothesy [5] are introduced. These approaches have been implemented on selected images from CASIA IRIS V3 [6] iris database. All iris authentication methods require the input iris strip as an iris template to be matched against a large number of iris templates in the database.

This paper is organized as follows. Section 2 discusses calculation of fractal dimension using box-counting approaches. Section 3 focus on a new information theoretic approach. Section 4 describes topothesy. Section 5 reports and discusses our experimental results. Section 6 offers a brief conclusion.

2 Estimating fractal dimension using Box-Counting approaches.

2.1 2-D Fractal calculation using Box-Counting on segmented images

The iris image (Fig. 1a.) is first pre-processed initially and then it is divided into a grid of blocks, on which the Sobel operator [8,9,10] is applied. If there are edges in two consecutive blocks, they may not be well connected. To take care of the continuity of edges, the Sobel operator is applied on two blocks together. Following this, an OR operation is performed between the two regions: one consisting of two separate edge images together and another single edge image comprising two blocks. The edge map thus obtained is shown in Fig. 1b. After getting the segmented image, the Box-counting method (explained below) [7] is applied on each block to determine the fractal dimension or Hausdorff dimension of the curve lying in the corresponding block of the grid.

The Box Counting Method is a way of sampling the thresholded regions of an image to find the rate of change in complexity with scale, as well as the

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measure of heterogeneity. It successively divides the image (an individual image block, in this case) into grids of different sizes. Each grid is a composed of a definite number of boxes, which corresponds to the scaling factor used. The number of boxes, falling into the thresholded region is counted for each grid.

As segmented images are the input in this method, the background is considered black and the foreground white (texture in this case). If N(□) is the total number of boxes with detail and if □ is the scale to obtain the corresponding grid, the fractal dimension d is given by:

$$D = \lim_{\epsilon \to 0} \frac{\log(N(\varepsilon))}{\log(1/\epsilon)} \quad (1)$$

which is derived from the scaling rule as:

$$N \propto (1/\epsilon)^d \quad (2)$$

If we consider different values of N at different scaling factorsϵ_i as N_i, the fractal dimension D is derived from the slope of $\log(N_i)$ Vs. $\log(1/\epsilon_i)$.

2.2 3D Fractal dimension using Differential Box-counting [2][3]

In the 3D Differential Box-Counting (3D DBC)method, the iris image is considered as occupying 3D space, with the pixel coordinates accounting for the two dimensions and the gray level of the pixels as the third dimension. This is an extension of the approach explained in Section 2.1 and hence the name. The *xy*-plane is partitioned into grids of size $s \times s$ according to the scaling factor r. Each grid contains a column of boxes, each of which measures $s \times s \times s$. Assuming that the minimum and the maximum gray levels in the $(i,j)^{th}$sub-region of a grid fall into the k^{th} and l^{th} boxes, respectively, the contribution that these make in the $(i,j)^{th}$ sub-region of grid is defined as :

$$n_r(i\,,j) = l - k + 1 \quad (3)$$

The total contribution from all the sub-regions of a grid in a block of the image corresponding to scaling factor r is therefore

$$N_r = \sum_{i\,,j} n_r(i\,,j) \quad (4)$$

For different values of r, we get different values of N_r which obey the regression equation,

$$\log(N_r) = \log(K) + D\log(1/r) \quad (5)$$

Here D represents the fractal dimension calculated using 3D DBC approach.

3. Estimating Fractal dimension using *Information Theoretic Approaches*

Section 2 had discussed the traditional tools for calculation of fractal dimensions. Here we introduce the concept of *Information set* which forms the basis of the derivation of *Information Theoretic Approach* based features.

3.1 Definition of Information Set [11]

Consider a fuzzy set $I = \{I(i,j)\}$ constructed from gray levels I in a window. This step is basically the granularization of a dataset, which is an image here. If an attribute or property in the window follows a distribution, it is easy to fit a membership function or at least an approximating function describing the distribution. Then the attributes or elements of the fuzzy set are represented by the membership function grades. It can be proved that the product of gray levels {I(i, j)} or information source values and their membership grades $\{\mu_{ij}\}$, are the information values, which constitute the *information set* defined as

$$H = \{\mu_{ij} I(i,j)\} \quad (6)$$

This relation is derived from the Hanman-Anirban entropy function [15] when both the exponential function and the Gaussian function are used. The exponential function used here is given by

$$\mu_{ij}^{e} = e^{-\left\{\frac{|I(i,j)-I(ref)|}{f_h^2}\right\}} \quad (7)$$

Wheref_h^2 is a fuzzifier defined by

$$f_h^2(ref) = \sum_{j=1}^{W}\sum_{i=1}^{W} \frac{(I(ref)-I(i,j))^4}{(I(ref)-I(i,j))^2} \quad (8)$$

In view of (7), the information set becomes $\{\mu_{ij}^{e} I(i,j)\}$. The unknown parameters in (7) are now detailed. The fuzzifier (f_h^2) is a kind of spread function, as it gives the spread of the attribute values with respect to the chosen reference, I(ref). It is the reference gray level in an image, which can be taken as the maximum gray level or median. In the present work, the maximum gray level is considered.

3.1.1 *Effective Information Source*

The Centroidal approach is used to compute the *effective information source* which is the information based feature obtained from the information values $\{\mu_{ij}^{e} I(i,j)\}$ of ξ sized block of image as

$$\vec{I}(\xi) = \frac{\sum_j \sum_i \mu_{ij}^{e} I(i,j)}{\sum_j \sum_i \mu_{ij}^{e}} \quad (9)$$

In addition to this, we will also explore the fractal based feature. A brief description of fractal dimension is in order.

Fractal Dimension from information theoretic approaches: Patterns in nature possess self similarity as can be observed from the leaves of trees, which retain the same shape even though they may be of different sizes. This property of iris images has been utilized in this approach by examining several sub images for the presence of similar texture patterns. While iris textures in different sub images are random in nature, it is still possible to compute the fractal dimension underlining these images

Initially the images are partitioned into blocksof the same size. Then sub blocks of different scales within a given window are selected. Given a size of sub block given by ξ, the size of the corresponding block can be calculated using a scale factor s leading to $s\xi$. If the sub blocks satisfy the self-similarity property in some attribute, then it is possible to compute the fractal dimension which underlies this block. To measure the self similarity by way of fractal dimension, let us consider a block having n number of sub blocksizes associated with n scale factors, where the j^{th} scaling factor is represented by s_j. Within the given block, there will be m_{s_j} sub blocks corresponding to the scaling factor s_j. $\vec{I}_k(\xi)$ is the Effective Information Source value associated with k^{th} sub block, which is the outcome of varying the scaling factor s_j and $\vec{I}(s_j\xi)$ is the block, whose size is $s_j\xi$.The fractal dimension is computed as,

$$R_c = \sum_{j=1}^{n} \sum_{k=1}^{m_{s_j}} \ln_{s_j}\left(\frac{\vec{I}(s_j\xi)}{\vec{I}_k(\xi)}\right) \quad (10)$$

3.1.2 *Hanman Transform.*

This transformation is motivated from the fact that any information source (text, image or video) must be weighed as a function of the information. Note that the information results from an agent who gives the information source a grade. This transform is derived from the Hanman-Anirban entropy function in [15]

$$H(p) = \sum_{i=1}^{n} p_i e^{-(ap_i^3+bp_i^2+cp_i+d)} \quad (11)$$

By an appropriate choice of parameters a=b=d=0 and c=$\mu_{ij}^{e}/I_{\max}$,$p_i = I(i,j)$in (11) we get the expression for the transform as

$$\overleftarrow{I}(\xi) = \sum_j \sum_i I(i,j) e^{-(\mu_{ij} I(i,j)/I_{\max})} \quad (12)$$

Several applications of this transform include: creation of new features, evaluation of quality of signals, image processing and video processing to name a few.

$\overleftarrow{I}(\xi)$is the Hanman Transform corresponding to the particular block of sizeξ. The information is obtained as the sum of the matrix elements. Replacing $\vec{I}(\xi)$ by $\overleftarrow{I}(\xi)$ in (10), we get the expression for fractal dimension as

$$R_c = \sum_{j=1}^{n} \sum_{k=1}^{m_{s_j}} \ln_{s_j}\left(\frac{\overleftarrow{I}(s_j\xi)}{\overleftarrow{I}_k(\xi)}\right) \quad (13)$$

4. Estimating features using Topothesy

There have been numerous attempts at using fractals for quantifying the surface finish of engineering materials. In these applications, topothesy is recommended as a measure of the fractal nature of the surface, which is least affected by scale variations in measurement[19]. As a result in tribology and surface metrology, topothesy is considered as a better measure of quantifying the rough surfaces. Topothesy represents the horizontal distance between two points on a measured surface corresponding to an average slope of one radian and has a dimensional unit of distance[16][17]. Fractals for engineering surfaces can be characterized by two parameters, the fractal dimension D and the topothesyΛ. These parameters have been determined experimentally by measuring the slope and intercept of a logarithmic plot of the structure function(S(τ)). We will now discuss the structure function.

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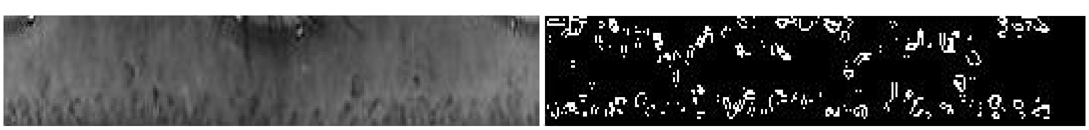

(a) (b)

Fig.1. Iris strip segmentation (a) Iris strip (b) Segmented iris strip.

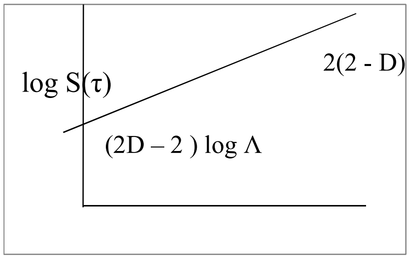

Figure 2. Calculation of fractal dimension and topothesy using the structure function[17].

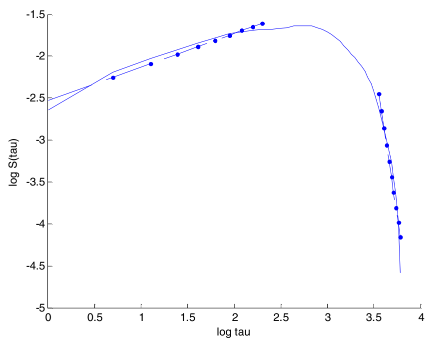

Figure 3. Tangents are drawn on best fit hyperbola, which is obtained from $S(\tau)$.

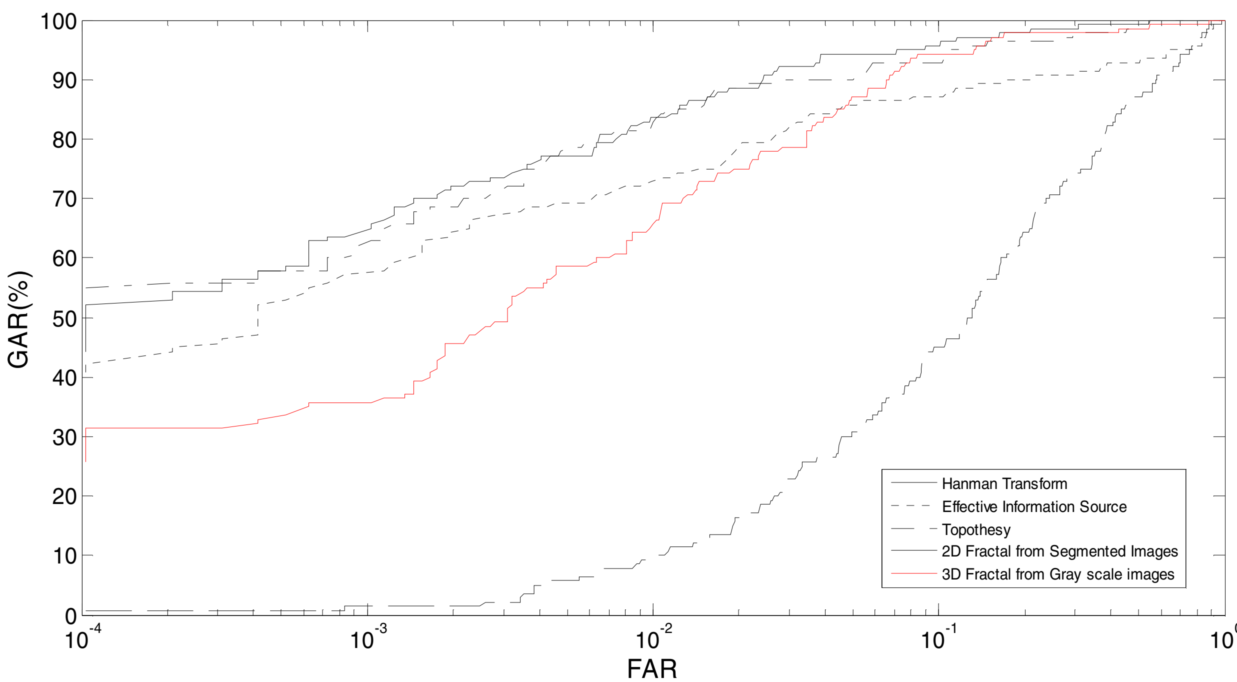

Figure 4. ROC plot for fractal features from different approaches

Table 1 : Fractal features derived from different approaches and their respective accuracies with different classifiers

Classifier Features	Bayesian	SVM		
		Linear	2^{nd} Degree Polynomial	3 rd Degree polynomial

2D DBC	37.9	31.4	32.9	32.1
3D DBC	80	80.7	82.1	78.6
Effective Information Source	**90**	89.3	82.1	82.9
Fuzzy Fractal with Hanman Transform	**91.4**	87.9	87.1	90
Topothesy	27.9	87.9	**92.9**	85

4.1 The Structure Function [5][18]

Structure function is a measure of the frequency variation in a signal. A structure function (SF) for a particular 1D discrete function *z(x)* and for a particular value of delay $\tau(\tau\epsilon \mathrm{Z}, 1 < \tau < m)$ is represented as

$$S(\tau) = \frac{1}{m-\tau}\sum_{i=1}^{m-\tau}\{z(i) - z(i+\tau)\}, \quad (14)$$

where, *m* is the number of data points available. Also, the structure function is a function of the fractal parameters [12]:

$$S(\tau) = \Lambda^{(2D-2)}\tau^{2(2-D)} \quad (15)$$

Hence, the parameters may be determined directly from the slope and intercept of a logarithmic plot (Fig.2). The plot is made by considering the structure function as 1D (τ). As the images involve two dimensions τ_x and τ_y, the structure function in 1D is changed to 2D as:

$$S(\tau_x, \tau_y) = \frac{1}{(m-\tau_x)(n-\tau_y)}\sum_{i=1}^{m-\tau_x}\sum_{j=1}^{n-\tau_y}\{ z(i,j) - z(i+\tau_x, j+\tau_y)\} \quad (16)$$

where, the size of image is m×n.

The values of structure function at τ_x=τ_y is plotted. A hyperbola is fitted to this plot and then it is approximated by asymptotes. The y-intercepts and slopes of the two tangents drawn on the resultant hyperbola, can be used to find the fractal dimension D and topothesy Λ using (16). The hyperbola and the asymptotes fitted to it are shown in Fig. 3 for a sample iris texture image. The images are divided into windows and the corresponding structure function is framed. The values of the structure function are plotted and asymptotes of the resulting hyperbolic curve are obtained. The slope of the asymptotes will provide us with the value of fractal dimension (D). The y-intercept gives the value of Topothesy(Λ). The features considered for experimentation are fractal dimension and topothesy calculated from this method concatenated to one another.

5. Results and Discussions

CASIA IRIS V3 Interval images were used for the experimental study. In this 5 images were selected for each person, of which 3 images were used for training and the rest for testing. The iris images were converted into rectangular iris strips. These were divided into blocks and the Information set based features were extracted using the new methods. The 3D DBC method was incorporated for comparison. The information set based methods, the effective information source and Hanman Transform features demonstrated very good performance. A topothesy based approach also yielded considerably good results. The classification was done using Bayesian and SVM classifier from PR tools [13].
The dimensionality was 60 for fractal dimensions calculated based on Effective information based feature and Hanman Transform. Topothesy and 2-D DBC had a dimensionality of 270 and 28 respectively. Figure 4 shows the ROC plot of fractals from the tested approaches. The results obtained clearly show a marked improvement over the coarse classification results presented in [2], where the classification was done for four categories only. The current study was conducted on iris images from 70 persons. Efforts are being made to apply and test the features on a bigger database and other biometric traits.

6. Conclusions

A comparative study of different methods of computing fractal dimensions from the iris texture was the main objective of this work. Information set based approach was tried for the deduction of fractal dimension. Topothesy based methods, which are prevalent in the domain of surface finish, were employed to enable the characterization of iris texture images for personal authentication. Tests with conventional classifiers give a maximum accuracy of 92.9% on the topothesy based features and 91.4% on the Effective Information Source based features. These two feature types are useful for the finer classification whereas 3D DBC for the coarse classification. As these features are meant for the characterization of texture, one can use them for other textures in biometrics like finger knuckle prints [20] and palm prints.

This study shows that the classical fractal methods such as Box counting fail to represent the iris texture effectively due to the lack of absolute self similarity within iris; but the features based on topothesy seem to be adapted to iris texture as they are well established for the surface characterization of anisotropic surfaces. The Information set based fractal measure also has similar characteristics. An exhaustive study is required to indicate its suitability to different textures.

This study demonstrates that iris texture exhibits the anisotropy property along with fractal nature. This is borne out from the visual appearance of iris texture, which shows some similarities not the absolute similarity in the entire region of iris of an individual. Thus the current study is only a starting point in the fractal based texture representation for the biometric textures. Some preliminary results in this direction are presented in [20].

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